In Exercises 13 through 16 find the product **AB**.

13. $\mathbf{A} = [1, \ 4, \ -2, \ 3]$, $\mathbf{B} = [2, \ 1, \ -1, \ 2]^T$.

14. $\mathbf{A} = [2, \ 3, \ 1, \ 4]$, $\mathbf{B} = [3, \ 1, \ 1, \ 3]^T$.

15. $\mathbf{A} = [1, \ 4, \ 3, \ 7, \ 5]$, $\mathbf{B} = [2, \ 2, \ -1, \ -1, \ 3]^T$.

16. $\mathbf{A} = [1, \ 3, \ -1, \ 2, \ 0]$, $\mathbf{B} = [-1, \ 2, \ 13, \ 4, \ 1]^T$.

In Exercises 17 through 22 find the product **AB** and, when it exists, the product **BA**.

17. $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 1 \end{bmatrix}$.

18. $\mathbf{A} = [1, 4, 6, -7]$, $\mathbf{B} = [2, 3, -2, 3]^T$.

19. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & -5 \\ 7 & 2 & 0 \end{bmatrix}$.

20. $\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 9 & -1 & 4 \\ 1 & 6 & -2 \\ 2 & 2 & 3 \end{bmatrix}$.

21. $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 2 & 2 & 6 \\ 1 & 5 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 0 & 4 \\ 1 & 4 & 7 \end{bmatrix}$. D.Ú.

22. $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 1 & 4 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 6 & -2 \\ -1 & 4 \end{bmatrix}$.

23. Given

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & -3 \\ 5 & 1 & 4 \\ -3 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & 6 \\ 1 & 6 & 3 \end{bmatrix},$$

show that $(\mathbf{AB})^T = \mathbf{BA}$.

In Exercises 24 through 28 write the given systems of equations in the matrix form $\mathbf{Ax} = \mathbf{b}$, where **A** is the coefficient matrix, **x** is the vector of unknowns, and **b** is the nonhomogeneous vector term.

24. $3x + 5y - 6z = 7$
$x - 7y + 4z = -3$
$2x + 4y - 5z = 4.$

25. $4u + 5v - w + 7z = 25$
$3u + 2v + 3z = 6$
$v + 6w - 7z = 0.$

26. $5x + 3y - 6z = 14$
$6x - 5y + 11z = 20$
$x - 4y + 3z = 2$
$9x - 3y + 2z = 35.$

27. $3x + 4y - 2z = \lambda x$
$2x - 7y + 6z = \lambda y$
$8x + 3y + 5z = \lambda z.$

28. $2x + 3y + 6z = \lambda(3x + 2y + 3z)$
$3x - 4y + 2z = \lambda(x - 5y + 2z)$
$4x + 9y + 2z = \lambda(x - 2y + 4z).$

29. If

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 6 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}, \quad \text{and}$$

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix},$$

solve for **X** given that

$$3\mathbf{X} + \mathbf{A} = \mathbf{A}^T\mathbf{B} - \mathbf{X} + 3\mathbf{B}.$$

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 1 \\ 3 & 0 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \quad \text{and}$$

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix},$$

solve for **X** given that

$$2\mathbf{AB}^T + \mathbf{X} - 2\mathbf{I} = 3\mathbf{X} + 4\mathbf{B} - 2\mathbf{A}.$$

31. Given that

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix},$$

show that

$$\mathbf{A}^3 - 9\mathbf{A}^2 + 18\mathbf{A} = \mathbf{0}.$$

32. Given that

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix},$$

show that

$$\mathbf{A}^3 + 2\mathbf{A}^2 - \mathbf{A} - 2\mathbf{I} = \mathbf{0}.$$

33. Prove the second result in Theorem 3.1 that $\mathbf{A(BC)} = \mathbf{(AB)C} = \mathbf{ABC}$.

34. Prove the third result in Theorem 3.1 that $(\lambda\mathbf{A})\mathbf{B} = \mathbf{A}(\lambda\mathbf{B}) = \lambda\mathbf{AB}$.

35. Prove the fourth result in Theorem 3.1 that $\mathbf{A(B + C)} = \mathbf{AB} + \mathbf{AC}$.

In Exercises 36 through 39 verify that $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$.

36. $\mathbf{A} = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 2 & 1 \end{bmatrix}$.

37. $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 & 3 \\ 1 & 6 & 2 & 1 \\ 1 & 1 & -2 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 5 \\ -1 & 3 & 2 \\ 1 & 7 & 3 \end{bmatrix}$.

38. $\mathbf{A} = \begin{bmatrix} 1 & 4 & 2 \\ 7 & 3 & -1 \\ 0 & 2 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 1 & -5 \\ 1 & 3 & 4 \\ 2 & 0 & 8 \end{bmatrix}$.

39. $\mathbf{A} = \begin{bmatrix} 1 & 4 & 6 & 2 \\ 2 & 1 & 4 & 1 \\ 3 & 0 & 0 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 4 \\ 2 & 2 & 5 \\ 1 & 1 & 1 \end{bmatrix}$.

40. Verify that $(\mathbf{ABC})^T = \mathbf{C}^T\mathbf{B}^T\mathbf{A}^T$ given that

$$\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -2 & 3 \\ 5 & 7 \end{bmatrix}.$$

41. Prove that if **D** is the $n \times n$ diagonal matrix

$$\mathbf{D} = \begin{bmatrix} k_1 & 0 & 0 & \cdots & 0 \\ 0 & k_2 & 0 & \cdots & 0 \\ 0 & 0 & k_3 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & k_n \end{bmatrix}, \quad \text{then}$$

$$\mathbf{D}^m = \begin{bmatrix} k_1^m & 0 & 0 & \cdots & 0 \\ 0 & k_2^m & 0 & \cdots & 0 \\ 0 & 0 & k_3^m & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & k_n^m \end{bmatrix},$$

where m is a positive integer.

Jeffrey, A., Advanced Engineering Mathematics, Ac. Press, 2001